Energy Transfer Processes in a Partially Ionized, Two-Temperature Gas

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The elementary situation of convective energy transfer between parallel plates at different temperatures is analyzed for partially ionized, nonreacting, monatomic hydrogen wherein the electrons may be at a temperature different from that of the heavier species. The gas conditions considered are such that an ion sheath, less than a mean free path in spatial extent, forms on the fully catalytic colder surface. The numerical results obtained indicate that 1) the gross features of the continuum solution are rather insensitive to the models of sheath and transition regions employed; 2) the energy transfer between ions and electrons due to the electric field external to the sheath is appreciable, particularly in its effect of reducing electron temperature; 3) thermal nonequilibrium effects on total heat transfer are not as great as might be anticipated because of the energy transfer between species by electric fields and the contribution of diffusional processes; and 4) magnetic field effects on total heat transfer are less than the effect on electronic thermal conductivity would indicate and are further lessened by thermal nonequilibrium effects.

Nomenclature

A_w = velocity ratio defining edge of transition region $v_{A,t}/v_{A,s}$
B = magnetic induction
C_j = mean thermal speed of species j
c_p = specific heat at constant pressure
D_{jk} = binary diffusion coefficient of mixture of species j and k
D_A = ambipolar diffusion coefficient ($D_A = 2D_{in}$)
E = electric intensity
e = magnitude of electronic charge
f = neutral fraction $= n_n/(n_i + n_n)$
F_w = diffusion flux of ions
j = current density
k = Boltzmann's constant
k_j = thermal conductivity of species j in the total mixture
L = distance between plates
Le = Lewis number, $\rho_r c_{pg} D_\infty / k_\infty$
m = particle mass
n = particle number density
p = total pressure of mixture
p_j = partial pressure of species j
Q_0 = factor for variation of ion-neutral diffusion coefficient, defined by Eq. (28)
Q = collision cross section
Sc = Schmidt number, $\mu_\infty/\rho_r D_\infty$
T = temperature
u, v, w = mass velocity of mixture
u_A, v_A, w_A = mass velocity of ions relative to the mixture
x, y, z = Cartesian components
α = current fraction, defined by Eq. (2)
β = Hall parameter $= (\omega\tau)$
γ = $e|\phi_{sw}|/kT_{e,s}$
γ_1 = $e|\phi_{tw}|/kT_{e,t}$
δ = $[e\rho(1 - f)/m_i]^2/\sigma$
δ_{en} = energy exchange effectiveness, defined by Eq. (29)
ϵ_n = negative ionization energy
λ_{jk} = coefficient of collisional energy transfer from species k to species j
ρ = mass density of mixture
σ = electrical conductivity
ϕ_{jk} = electrical potential between points j and k

Subscripts

c = conduction
d = diffusion
e = electrons
g, h = heavy particles
i = ions
n = neutral particles
r = quantity used for normalization
s = sheath edge
t = edge of transition region
w = cold surface
x, y, z = Cartesian components
∞ = hot surface

I. Introduction

The occurrence of an appreciable degree of ionization in a gas introduces some features quite different from those encountered in chemical dissociation primarily because of the presence of electrons. First, the extremely low mass of the electrons yields a species possessing a thermal conductivity that can be much greater than that of the other species present; second, the collisional energy-transfer processes between electrons and heavy particles are relatively slow, giving rise to the possible situation wherein the electrons may have a temperature much different from that of the heavy species, even though the energy-transfer processes are adequately described by continuum equations; third, the charged species are sensitive to electromagnetic fields, yielding a possible method of controlling the associated components of energy transfer and further yielding an additional mechanism for energy transfer between electrons and ions; fourth, when such a gas is in contact with a cold surface, a space charge sheath is formed which may appreciably affect the energy transfer to the surface. The physical situations in which these unique features of appreciably ionized gases are likely to manifest themselves are becoming more frequent, both in the laboratory and in practical applications. Therefore, it appears that investigations aimed toward obtaining a physical appreciation of these phenomena are appropriate; this view is supported by the recent appearance in the literature[1, 2] of two investigations in the general problem area where the electrons present in a gas are not thermally equilibrated with the heavy particles.

Camac and Kemp[1] have formulated the shock tube end wall boundary-layer problem in appreciably ionized argon, at conditions where the Debye length is less than the ion-neutral

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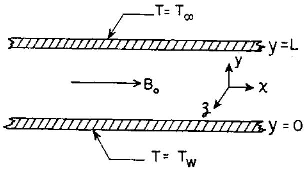

Fig. 1 Geometry of parallel-plate problem.

mean free path, and have obtained two numerical results; one for the limiting case of complete thermal equilibrium and the other for the limiting case of no collisional energy transfer between ions and electrons, both results being based upon the local similarity approximation. Although these results are closely related to a clearly physical situation, it is difficult to extract the roles of the various energy-transfer processes from only these results, and the interpretation is further complicated by the use of the local similarity approximation. For these reasons, the entirely different approach employed in the present investigation (described in the next paragraph), aimed specifically toward obtaining an understanding of the roles of the various energy-transfer processes, appears to have some merit, although the formulation of the basic equations and boundary conditions are closely related to that of Camac and Kemp. Chung and Mullen[2] have investigated thermal nonequilibrium effects in a stagnation-point boundary layer in a slightly ionized gas, with primary emphasis on electrical characteristics, rather than energy transfer. The major differences in the formulation of the basic equations and the cold surface boundary conditions utilized by Chung and Mullen and those to be employed here are that the former considers the sheath to be collision dominated and, in addition, incorporates the assumptions that the energy transfer between ions and electrons due to electric fields is negligible everywhere and that the diffusional velocities are proportional to gradients in mass fractions, rather than partial pressures. These latter assumptions are not justifiable in the general case.

As any detailed analysis of an appreciably ionized gas is necessarily complex, the situation considered here is the elementary one of a gas between parallel plates of different temperatures (to be described more fully in the succeeding section) for which results can be obtained without resorting to approximate techniques. The purpose of the investigation of this elementary situation is, in addition to presenting new computational results, to obtain a physical appreciation of the aforementioned features of ionized gases, which is not readily obtainable in previous work, in the belief that the results obtained can be used as a qualitative indication of the effects in more complicated flow situations (e.g., boundary layers) as well as a guide to permissible simplifications in these more complex problems. Specific emphasis is placed upon the effects on energy transfer to a cold surface, including the effects on the individual processes (thermal conduction and diffusional transport by both electrons and heavy particles) of the following phenomena: 1) the role of the space charge sheath (in which a continuum description is not strictly valid and which can be solved in only very crude ways) in determining the appropriate boundary conditions for the continuum equations: this follows closely the work of Camac and Kemp,[1] with some additional considerations of sheaths in which the ion temperature is not negligible, as well as sheaths in which the electron and ion flow processes are each considered adiabatic rather than spatially isothermal; 2) the presence of the transition region between the sheath and the region where the usual continuum equations are valid; 3) the contribution of the energy transfer between ions and electrons due to the electric field external to the sheath; 4) thermal nonequilibrium between electrons and heavy particles; and, finally, 5) the effect of a magnetic field parallel to a cold surface.

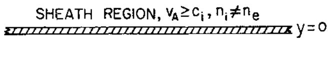

Fig. 2 Flow regimes near a cold surface.

II. Problem Formulation

2.1 Problem Definition

The physical situation considered here is that of partially ionized atomic hydrogen placed between two electrically conducting plates at different temperatures (see Fig. 1).† The steady-state solution is desired wherein all quantities are a function of the single spatial variable y only. It is further assumed that 1) there is no net mass transfer between the gas and the cold surface at $y = 0$, hence $v(0) = 0$; 2) the applied magnetic field is uniform and in the positive x direction; 3) the applied electric field vanishes in the x and z directions, hence $E_x = E_z = 0$; 4) the magnetic field due to currents existing in the gas is negligible (this can be shown to be the case at the conditions encountered here), hence, $B_z = B_y = 0$, $B_x = B_0$; 5) the total pressure gradient in the y direction is negligible [this is valid if $(\omega\tau)_e(\omega\tau)_i \ll 1$]. With these assumptions, the governing equations (Sec. 2.2) indicate that, if the magnetic field and a net current flow do not occur simultaneously, there is no net mass motion of the gas, and diffusion and current flow (if it exists) occur only in the direction normal to the plates. If the magnetic field and a net current flow normal to the plates exist simultaneously, then, in addition to diffusion normal to the plates, mass motion, current flow, and diffusion also occur in the z direction.

The conditions considered in the present investigation are such that near the cold surface, which is assumed fully catalytic to electron-ion recombination,‡ the Debye length is less than a heavy particle mean free path. In this case, it is well known that the following three rather distinct regions exist near the surface (Fig. 2): 1) away from the wall, the gas is quasi-neutral, the ion diffusion velocity is small, and the behavior of the gas in this region is described by continuum equations; 2) near the wall (but not adjacent to it), the gas remains quasi-neutral, but the ion diffusion velocity is comparable to the ion sound velocity and the usual equations are not valid; this region will be referred to here as the "transition" region; and 3) adjacent to the wall, a space charge sheath, on the order of a Debye length in spatial extent, exists wherein the gas is no longer quasi-neutral (in all cases considered here, the sheath is composed of excess ions, yielding an electric field that acts to repel electrons). A major difficulty exists in connection with the latter two regions in that, to obtain precise solutions, it is necessary to solve the Boltzmann (or Vlasov, as appropriate) equation for each species, and at present this does not constitute a tractable problem. The approach employed here, in this regard, is to obtain crude solutions in these regions, which relate the species' temperatures and velocities at the edge of the transition region to the known properties of the cold surface. Combined with the observation that the spatial extent of these regions is negligible compared to the plate

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† The hotter "plate" is actually best described as a control surface in this investigation in that the temperatures of the species and the mixture composition are prescribed here, thus permitting diffusion to occur across the surface and allowing a nontrivial steady-state solution to be obtained. Further, with this interpretation, the radiative properties of the hotter plate are then determined by the properties of the gas and the amount that exists above this surface.

‡ It is assumed that only monatomic hydrogen is produced by recombination at the surface. This may be unrealistic for hydrogen since some of the diatomic species may be formed, depending upon the catalytic properties of the surface. However, the results to be obtained are qualitatively similar for any monatomic gas

separation, these solutions are employed to provide boundary conditions at $y = 0$ for the continuum equations in an attempt to determine the extremes of behavior to be expected and the resultant effects upon the solution to these equations. The continuum equations are presented in Sec. 2.2, and a detailed formulation of the boundary conditions is presented in Sec. 2.4.

To conclude the problem definition, the following assumptions are employed in the derivation of the continuum equations presented in the next section: 1) the mixture remains quasi-neutral; 2) the ratio of electron to ion mass is negligible compared to unity; 3) the diffusional velocity of each species is negligible compared to its thermal (or sonic) velocity; 4) the temperatures of ionic and neutral species are equal; 5) energy transfer due to radiation is negligible; and 6) there exists no gas-phase chemical reaction. The first three assumptions are rather common, and merit no additional comment. Because of the essentially equal masses of the heavier species, assumption 4 must be valid in continuum flow. The validity of the last two assumptions depends in detail upon the gas conditions, surface properties, and the particular geometrical configuration contemplated. For radiative transfer considerations, it can be shown by utilizing the work of Bates and Kingston[13] that, for monatomic hydrogen at conditions of interest ($p \sim 0.1$–1.0 atm, $T \sim 15{,}000°$–$50{,}000°$K), the gas is transparent over the length in which appreciable thermal nonequilibrium is likely to occur; further, the emitted radiation from this depth is appreciably smaller than the convective energy transfer. For example, for hydrogen at 50,000°K and 1 atm, the length over which appreciable thermal nonequilibrium exists is approximately 0.4 mm, and the ratio of the convected energy transfer through this depth to a cold surface to the emitted radiation is approximately 150. Thus, if a boundary-layer situation is visualized, a depth of gas of approximately 6 cm is required before the total emitted radiation is equal to the convective energy transfer across a boundary layer of 0.4 mm thickness. For heavier gases, the relative importance of radiation would probably be somewhat greater.

The amount of gas-phase reaction is, of course, determined by the ratio of the residence time of a particle between the plates, which is governed by the diffusion process, to a characteristic reaction time. Gas-phase reaction in the presence of thermal nonequilibrium is somewhat more complex than that at thermal equilibrium since, in addition to the importance of the recombination mechanisms at lower temperatures near thermal equilibrium, the ionization mechanisms are likely to become important at low gas temperatures if the electron temperature is sufficiently high. A more detailed discussion of this matter is beyond the scope of the present work; however, it can be stated that, as pressure and/or plate spacing are decreased, the relative importance of gas-phase reaction decreases, and it is probable that conditions do exist in which they are unimportant. It should be emphasized at this point that the primary purpose of this investigation is to examine the nature of two temperature effects alone on convective energy transfer. As will be seen, this exhibits several important effects even in the absence of other complicating phenomena. Thus, although the matters of radiative transfer and gas-phase reaction are certainly legitimate areas for further investigation, since both can contribute importantly to the energy transfer between electrons and heavy particles, it is believed by this writer that the present investigation is justified by the merits of revealing important effects as well as providing a helpful guide in the investigation of these other phenomena.

2.2 Governing Continuum Equations

Utilizing the previous assumptions, the continuum equations for the parallel plate configuration can be written as follows. Conservation of mass§ requires $v = 0$; conservation of ions requires, after integration with respect to y,

$$\rho(1 - f)v_A = F_w = \text{const} \tag{1}$$

Conservation of current requires

$$j_y = j_w = \alpha \frac{e\rho(1 - f)}{m_i} v_A \tag{2}$$

which also serves to define the current fraction α. The z component of the momentum equation is the only nontrivial one:

$$(d/dy)[\mu_{in}(dw/dy)] = j_y B_0 \tag{3}$$

where μ_{in} is the viscosity of the heavy particle mixture (the viscosity due to electrons is negligible).

Since the primary case of interest here is the situation where the electron temperature is different from that of the heavy particles, two energy equations are required. For the heavy particles, one obtains

$$\frac{d}{dy}\left(-k_{in}\frac{dT_g}{dy}\right) = \frac{e\rho(1 - f)}{m_i} v_A E_y + \lambda_{eh}(T_e - T_g) \tag{4}$$

whereas the electron energy equation is

$$\frac{d}{dy}\left(-k_e \frac{dT_e}{dy}\right) = -\frac{e\rho(1 - f)}{m_i} v_A E_y + E_y j_y - \lambda_{eh}(T_e - T_g) + \frac{d}{dy}\left\{\left[\rho(1 - f)v_A - \frac{m_i}{e} j_y\right] c_{pg} T_e\right\} \tag{5}$$

where each species is assumed to behave as a perfect gas. It is pointed out that, in addition to the usual terms, these equations contain a term representing energy transfer between light and heavy species due to collisional processes $[\lambda_{el}(T_e - T_g)]$ and a term representing energy transfer between charged species due to motion in an electric field (first term on the right side of both equations).

The generalized form of Ohm's law yields $j_x = 0$ and

$$j_y \frac{(1 + \beta_e{}^2)}{\sigma} = E_y + wB_0 + \frac{m_i}{e\rho(1 - f)} \frac{dp_e}{dy} \tag{6}$$

$$j_z = \beta_e j_y \tag{7}$$

Similarly, the diffusion equation yields $u_A = 0$ and

$$\frac{2(1 - f)p}{D_A[1 + (1 - f)(T_e/T_g)]} v_A - f j_z B_0 = -\frac{d}{dy}(p_i + p_e) \tag{8}$$

$$\frac{2(1 - f)p}{D_A[1 + (1 - f)(T_e/T_g)]} w_A = -f j_y B_0 \tag{9}$$

It is noted that the scalar electrical conductivity in (6) can be related to the binary diffusion coefficients¶ by utilizing the diffusional relations presented by Hirschfelder, Curtiss, and Bird,[4] suitably modified by the fact that the momentum transfer between electrons and heavier species is governed by the electron temperature when the temperatures are unequal, and further neglecting the effects of thermal diffusion and strong electric and magnetic fields on the velocity distribution functions. The latter assumptions are not wholly justifiable in the general case, but their use will not alter the character of the results for effects of magnetic

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§ A more detailed development of the multidimensional form of the governing equations, consisting primarily of algebraic manipulation of the conservation relations for each species in which all terms are retained, can be found in Ref. 3; development of the present one-dimensional form can be found in Ref. 12. Unfortunately, space does not permit presentation of these developments here.

¶ The "binary" coefficients are here defined as the value of the diffusion coefficient in the binary mixture at the same number densities and temperatures as exist in the total mixture.

fields and electron temperature nonequilibrium on energy transfer (which is the purpose of the investigation), but will, of course, cause the quantitative results to be somewhat in error. These assumptions are further supported by the fact that, in the case treated here, the electronic Hall parameter, $\beta_e = (\omega\tau)_e$, is restricted to values less than six. The resulting relation is

$$\sigma^{-1} = \left(\frac{1}{n_e e}\right)^2 \left(\frac{f}{D_{en}} + \frac{1}{2D_{ie}}\right) \frac{p(1-f)}{(1-f+T_g/T_e)} \tag{10}$$

As has been previously noted by Kemp,[5] the ion diffusion velocity is proportional to partial pressure gradients in (8) rather than the more common case of concentration gradients. Actually, the former is the more fundamental form, with the importance of the distinction lying in the fact that this form reduces to the common one involving concentration gradients (expressed in terms of mass fractions) only if the temperatures of all species are equal.

Equations (1–8), with the thermal equations of state for each species, form a set of eleven equations in the unknowns T_e, T_g, v_A, f, ρ, j_y, j_z, w, E_y, p_i, p_e with (9) providing a determination of w_A (which will not be of interest here). To nondimensionalize these equations, it is first convenient to introduce the quantity $\delta = [e\rho(1-f)/m_i]^2/\sigma$ to replace the electrical conductivity and then to introduce the following normalization quantities: $L_r = L$, $\rho_r = 5p_\infty/2c_{pg}T_\infty$, $T_r = T_\infty$, $p_r = p_\infty$, $\mu_r = \mu_\infty$, $k_r = k_\infty$, $D_{A,r} = D_\infty$, $\delta_r = p_\infty/D_\infty$, $\lambda_r = k_\infty/L^2$, $E_r = kT_\infty/eL$, $B_r = B_0$, $v_{A,r} = D_\infty/L$, $w_r = E_r/B_r$, $F_{w,r} = \rho_r v_{A,r}$, and $j_r = (e/m_i)\rho_r v_{A,r}$. The subscript ∞ refers to conditions at the upper (hot) plate, and D_∞ is taken as the value of D_A at p_∞ and T_∞, assuming only a trace of ionization. The dimensionless parameters $Le = \rho_r c_{pg} D_\infty / k_\infty$, $Sc = \mu_\infty/\rho_r D_\infty$, and $\psi = e^2 B_0^2 L^2 / m_i k T_\infty$ arise as a consequence of the normalization scheme. The preceding equations, in terms of the dimensionless variables, become (after some manipulation)**

$$\rho(1-f)v_A = F_w \tag{11}$$

$$\frac{d}{dy}\left(\mu_{in}\frac{dw}{dy}\right) = \frac{\psi}{Sc}\,\alpha F_w \tag{12}$$

$$\frac{d}{dy}\left(-k_{in}\frac{dT_g}{dy}\right) = \frac{2}{5}(Le)\left[\alpha\delta(1+\beta_e{}^2)v_A{}^2 - F_w w - F_w v_A \frac{d}{dy}\left(\frac{T_e}{v_A}\right)\right] + \lambda_{eh}(T_e - T_g) \tag{13}$$

$$\frac{d}{dy}\left(-k_e\frac{dT_e}{dy}\right) = -\frac{2}{5}(Le)(1-\alpha)\left[\alpha\delta(1+\beta_e{}^2)v_A{}^2 - F_w w - F_w v_A \frac{d}{dy}\left(\frac{T_e}{v_A}\right)\right] - (Le)(1-\alpha)F_w\frac{dT_e}{dy} - \lambda_{eh}(T_e - T_g) \tag{14}$$

$$v_A = \frac{D_A}{2\rho(1-f)T_g(1-\beta_e\theta)}\,\frac{dg}{dy} \tag{15}$$

$$E_y = \frac{\alpha\delta}{F_w}(1+\beta_e{}^2)v_A{}^2 - w - v_A\frac{d}{dy}\left(\frac{T_e}{v_A}\right) \tag{16}$$

where

$$\rho = [T_g + (1-f)T_e]^{-1} \tag{17}$$

$$\theta = (eB_0 D_\infty/kT_\infty)\alpha D_A f/2T_g \tag{18}$$

$$g = f/[1 + (1-f)(T_e/T_g)] \tag{19}$$

In this form, Eqs. (12–15) represent a coupled set of four nonlinear differential equations in the variables (for example) T_e, T_g, w, and v_A.

2.3 Transport Properties††

To obtain the values of the various transport properties of the mixture, it is necessary to obtain the properties of the pure species‡‡ and then, in the case of viscosity and thermal conductivity, to compute the mixture properties by standard (although approximate) mixture rules. It should be remarked that experimental determination of these values is rather sparse and often nonexistent at the temperatures of interest, hence the following theoretical predictions are subject to error. Further, these theoretical predictions may not be the best available; however, it is believed that all significant qualitative trends are faithfully reproduced, which is adequate for the present purposes.

In this connection, it is pointed out that, although the properties are computed for partially ionized monatomic hydrogen (which is, in a sense, unrealistic because of the possible presence of H_2), these properties are qualitatively similar for any partially ionized monatomic gas, and hence some insight into the behavior of these other gases can be gained from the results for hydrogen. To elaborate somewhat on this remark, if it is desired to extrapolate the results obtained for hydrogen to some other gas, the features of the transport properties which must be accounted for are the change in characteristic magnitudes and the change in temperature-composition dependence.§§ Of these two features, the change in characteristic magnitudes can always be accounted for by an appropriate nondimensionalization scheme and hence causes no difficulty. If, however, the change in temperature-composition dependence of a particular property is markedly different in two gases, there exists no way to extrapolate the results from one gas to the other. Fortunately, this is not the case in the transport properties of hydrogen as computed here and the transport properties of the noble gases, since the relevant particle cross sections are not significantly different (with the single exception of the electron-neutral cross section, which does not play an important role at the gas conditions of interest). For example, if the thermal conductivities of the electronic component and the ion-neutral component and the ion-neutral diffusion coefficient as computed by Camac and Kemp[1] for argon are multiplied by one, ten, and three, respectively, it is found that the results are very close to the corresponding properties as computed here for monatomic hydrogen, with no qualitative change in temperature-composition dependence. Thus, it is expected that the results obtained in the present investigation can be qualitatively applied to argon, provided that the change in characteristic magnitudes are accounted for by nondimensionalization. Accordingly, the following theoretical predictions are presented without further apology.

The transport properties of a mixture of equal numbers of ions and electrons have been computed by Braginskii,[6] with the following results¶¶:

$$(\mu_i)_p = 0.246\left(\frac{m_i}{k}\right)(k_i)_p \doteq 0.18\left(\frac{2kT_g}{e^2}\right)^2\left(\frac{kT_g}{\pi}\right)^{1/2}\frac{m_i{}^{1/2}}{\ln\Lambda_i} \tag{20}$$

** Hereafter, dimensional quantities will be denoted by a prime, unless otherwise specified.

†† All quantities in this section are dimensional.

‡‡ As the transport properties of charged species are most readily available in terms of a fully ionized gas, the subsequent discussion considers a mixture of equal numbers of ions and electrons as a pure species.

§§ Any transport property q can be written as $q = q_r F(T_j, n_j)$, where q_r is the value of the transport property at some point in the flow. In the present terminology, q_r is referred to as the characteristic magnitude and F as the temperature-composition dependence.

¶¶ The notation employed is that a quantity enclosed in parentheses denotes a property of an individual species, the inner subscript referring to that species and the outer subscripts referring to the number of species present: i, e, n = ions, electrons, or neutrals, only, respectively; p = ions and electrons; no outer subscript indicates all species are present.

$$(k_e)_{p\|} = 0.419k\left(\frac{2kT_e}{e^2}\right)^2\left(\frac{kT_e}{\pi}\right)^{1/2}\frac{1}{m_i^{1/2}\ln\Lambda_e} \qquad (21)$$

$$D_{ie} = 0.293\frac{(kT_e)^{5/2}}{n_e m_e^{1/2} e^4 p_e \ln\Lambda_e} \qquad (22)$$

$$\lambda_{ie} = 3m_e n_e k/m_i\tau_{ei} \qquad (23)$$

where the subscript $\|$ refers to the direction parallel to the magnetic field and

$$\ln\Lambda_{i,e} = (3/2e^3)(k^3T_{g,\,e}{}^3/\pi n_{i,e})^{1/2} \qquad (24)$$

$$\tau_{ei} = 3m_e^{1/2}(kT_e)^{3/2}/4(2\pi)^{1/2}e^4 n_e \ln\Lambda_e \qquad (25)$$

Reasonable approximations of the results of Vanderslice, et al.[7] for the viscosity and thermal conductivity of atomic hydrogen are

$$(\mu_n)_n = 1.03 \times 10^{-8}T_g \text{ (kg/m-sec)} \qquad (26)$$

$$(k_n)_n = 3.20 \times 10^{-4}T_g \text{ (joules/m-sec-°K)} \qquad (27)$$

The value of the ion-neutral binary diffusion coefficient is obtained from the data of Brown[8] for helium and from assuming that the coefficient varies as the inverse square root of the atomic mass, with the result

$$D_{in} = 7.60 \times 10^{-10}T_g{}^2/p_{in}Q_0 \text{ (m}^2\text{/sec)} \qquad (28)$$

where p_{in} denotes the partial pressure (in atmospheres) of the ion-neutral mixture and $Q_0 = 1$. As this result is subject to considerable error, the factor Q_0 is included as a simple method for investigating possible changes in D_{in}.

For want of other data, the electron-neutral cross section is assumed to be $Q_{en} = 10^{-19}$ m^2, with the corresponding mean free time between collisions of $\tau_{en} = (n_nQ_{en}C_e)^{-1}$. The resulting energy exchange coefficient is

$$\lambda_{en} = \left(\frac{3}{2}k\right)\left(2\frac{m_e}{m_n}\right)\left(\frac{n_e}{\tau_{en}}\right)\delta_{en} \qquad (29)$$

where $\delta_{en} = 1$ is a good approximation for the effectiveness of a collision in transferring energy in monatomic gases. The electron-neutral diffusion coefficient is

$$D_{en} = 2.52 \times 10^{-6}T_e{}^{1/2}T_g/p_{in} \text{ (m}^2\text{/sec)} \qquad (30)$$

where p_{in} is in atmospheres.

The properties of the total mixture can be computed from the properties of the pure species by standard mean free path considerations proposed by Fay[9] and will not be discussed further here. The results are as follows: for the viscosity of the ion-neutral component,

$$\mu_{in} = (\mu_n)_n\left[1 + 1.35\frac{(1-f)}{f}Q_0\right]^{-1} + (\mu_i)_p\left[1 + (3.8 \times 10^{-8})\left(\frac{f}{1-f}\right)\frac{Q_0T_g{}^{3/2}}{\ln\Lambda_i}\right]^{-1} \qquad (31)$$

for the thermal conductivity of the ion-neutral components,

$$k_{in} = (k_{in})_n\left[1 + 1.35\left(\frac{1-f}{f}\right)Q_0\right]^{-1} + (k_i)_p\left[1 + (3.8 \times 10^{-8})\left(\frac{f}{1-f}\right)\frac{Q_0T_g{}^{3/2}}{\ln\Lambda_i}\right]^{-1} \qquad (32)$$

for the thermal conductivity of the electrons,

$$k_{e\|} = (k_e)_{p\|}\left[1 + (2.0 \times 10^{-10})\left(\frac{f}{1-f}\right)\frac{T_e{}^2}{\ln\Lambda_e}\right]^{-1} \qquad (33)$$

for the total energy exchange coefficient,

$$\lambda_{eh} = \lambda_{ei} + \lambda_{en} \qquad (34)$$

and for the electronic Hall parameter,

$$(\omega\tau)_e = (3.92 \times 10^{-12})\left(\frac{B}{p}\right)T_e{}^{5/2}\left(\frac{T_g}{T_e}\right) \times \frac{1 + (1-f)(T_e/T_g)}{(1-f)\ln\Lambda_e + (2.0 \times 10^{-10})fT_e{}^2} \qquad (35)$$

where B is in webers per square meter, p in atmospheres, and T in degrees Kelvin. Finally, the thermal conductivity of the electrons transverse to the magnetic field (the only direction of interest here) is taken as

$$k_e = [1 + (\omega\tau)_e{}^2]_{-1}k_{e\|} \qquad (36)$$

It should be mentioned that the ionic contributions to the thermal conductivity and viscosity, as well as the contributions of electron-neutral interaction to energy exchange and to the value of the Hall parameter, are essentially negligible at the conditions encountered in the present investigation.

2.4 Boundary Conditions

At the conditions of interest here, the most appropriate analysis of the sheath to date has been given by Schulz and Brown.[10] Their analysis incorporates the assumptions that the electrons are isothermal in the sheath and that the directed energy of the ions is much greater than the thermal energy; solution of the continuity and momentum equations (in the absence of transport effects), in conjunction with Poisson's equation, yields the result that the ion diffusion velocity at the sheath edge must satisfy the inequality (in dimensional form)

$$v_{A,\,s}{}^2 \geq kT_{e,\,s}/m_i \qquad (36a)$$

and the corresponding potential drop across the sheath satisfies

$$\gamma \equiv \frac{e|\phi_{sw}|}{kT_{e,\,s}} \leq \ln\left(\frac{C_e}{4|v_{A,\,s}|(1-\alpha)}\right) = \ln\frac{(m_i/2\pi m_e)^{1/2}}{(1-\alpha)} \qquad (36b)$$

where α is the ratio of the net current flow to the wall to the ion current (the analysis being restricted to $|\alpha| \lesssim 1$). Some elementary extensions of this solution which indicate the effect of other plausible assumptions are as follows. First, if the ion thermal energy is not neglected and both electrons and ions are assumed isothermal in the sheath, the corresponding results are

$$v_{A,\,s}{}^2 \geq (k/m_i)(T_{e,\,s} + T_{g,\,s}) \qquad (37a)$$

$$\gamma \equiv \frac{e|\phi_{sw}|}{kT_{e,\,s}} \leq \ln\frac{(m_i/2\pi m_e)^{1/2}}{(1-\alpha)(1 + T_{g,\,s}/T_{e,\,s})^{1/2}} \qquad (37b)$$

Similarly, if both electrons and ions are assumed to be adiabatic in the sheath, the results become

$$v_{A,\,s}{}^2 \geq \tfrac{5}{3}(k/m_i)(T_{e,\,s} + T_{g,\,s}) \qquad (38a)$$

$$\gamma \equiv \frac{e|\phi_{sw}|}{kT_{e,\,s}} \leq \ln\frac{(3m_i/10\pi m_e)^{1/2}}{(1-\alpha)(1 + T_{g,\,s}/T_{e,\,s})^{1/2}} \qquad (38b)$$

At the sheath edge then, the appropriate boundary conditions on the ion temperature and ion diffusion velocity are that $T_g = T_w$ (since the ions are in thermal equilibrium with the neutrals) and $v_A = v_{A,\,s}$,* respectively. The boundary condition relevant to the electron temperature is obtained by calculating the energy flux across the sheath edge carried by the electrons

$$q_{ew}' = \rho_{i,\,s}v_{A,\,s}(1-\alpha)(2+\gamma)(\tfrac{2}{5}c_{pg}T_{e,\,s}) \qquad (39)$$

which is the result of Jukes[11] quoted by Camac and Kemp[1] and may be obtained by assuming the electron velocity dis-

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* The equalities in the sheath relations are assumed to hold.[10]

tribution to be Maxwellian at the sheath edge and computing the energy flux carried toward the wall by electrons possessing sufficient energy to penetrate the sheath potential. It should be pointed out that, although only a very small fraction of the electrons incident on the sheath edge succeeds in penetrating it, this small fraction originates in the high energy portion of the Maxwellian distribution and hence carries a significant amount of energy. Further, the energy flux by electrons across the sheath edge does not represent the energy flux by electrons to the wall, since most of this energy is transferred to the ions in the sheath (via electric fields).

The problem remains to relate the quantities T_e, T_g, and v_A at the edge of the transition region to relevant properties at the sheath edge. In the work of Camac and Kemp,[1] the transition region was neglected, so that the boundary conditions for the continuum equations became $T_g = T_{g,\,s} = T_w, v_A = v_{A,\,s}$ and

$$[-k_e(dT_e/dy) + \rho_i v_A(1 - \alpha)c_{pg}T_{e,\,s}]_s = [\rho_i v_A(1 - \alpha)(2 + \gamma)(\tfrac{2}{5}c_{pg}T_e)]_s \quad (40)$$

where $v_{A,\,s}$ and γ were given by Eqs. (36). Boundary conditions of this type will also be considered here. In addition, it is proposed that the transition region is of very small spatial extent and therefore may be treated in a way analogous to the sheath treatment. The edge of the transition region will be defined by the relation $v_{A,\,t}/v_{A,\,s} = A_w$; that is, the ion diffusion velocity at the transition edge is some fraction of the velocity at the sheath edge. Then the continuity, momentum, and energy equations for ions and electrons in the transition region (where $n_i = n_e$ and transport effects are again neglected) may be solved with the same assumptions as led to Eqs. (36–38), respectively, to yield for the potential difference across both the sheath and transition region:

$$\gamma_1 \equiv \frac{e|\phi_{tw}|}{kT_{e,\,t}} \leq \ln\frac{(m_i/2\pi m_e)^{1/2}}{(1 - \alpha)} + \frac{1}{2}(1 - A_w^2) \quad (41)$$

for the cold ion, isothermal electron case;

$$\gamma_1 \leq \ln\frac{(m_i/2\pi m_e)^{1/2}}{(1 - \alpha)(1 + T_{g,\,t}/T_{e,\,t})^{1/2}} + \frac{1}{2}(1 - A_w^2) \quad (42)$$

for the isothermal ion and electron case†; and

$$\gamma_1 \leq \frac{3}{4}\left(1 + \frac{A_w^2}{4 - A_w^2}\right)\ln\frac{(3m_i/10\pi m_e)^{1/2}}{(1 - \alpha)(1 + T_{g,\,t}/T_{e,\,t})^{1/2}} + \frac{5(1 - A_w^2)}{2(4 - A_w^2)} \quad (43)$$

for the adiabatic electron and ion case. The boundary conditions at the edge of the transition region become then $T_{g,\,t} = T_w$ (the ions are still assumed to be in thermal equilibrium with the neutrals, and the transition region is assumed sufficiently small so that the temperature is essentially the wall temperature), $v_{A,\,t} = A_w v_{A,\,s}$, and the energy balance for electrons yields a relation identical to (40) with γ replaced by γ_1, if (41) or (42) apply, whereas if (43) applies, γ in (40) should be replaced by

$$\gamma_2 = \frac{1 - A_w^2}{2(4 - A_w^2)} + \frac{3}{4}\left(1 + \frac{A_w^2}{4 - A_w^2}\right)\ln\frac{(3m_i/10\pi m_e)^{1/2}}{(1 - \alpha)(1 + T_{g,\,t}/T_{e,\,t})^{1/2}} \quad (44)$$

In dimensionless form, (40) becomes, at $y \cong 0$,

$$k_e(dT_e/dy) = F_w(Le)(1 - \alpha)(0.2 - 0.4\gamma_t)T_e$$

The heat transfer to the cold surface is given by (in dimensionless form)

$$q_w = q_{ci} + q_{di} + q_{ce} + q_{de} + q_{is}$$

where the transfer by conduction due to ions and neutrals is

$$q_{ci} = [k_{in}(dT_g/dy)]_{y=0}$$

the transfer by diffusion of recombination energy is

$$q_{di} = -F_w(Le)(-\epsilon_n)$$

the transfer by conduction in the electrons is

$$q_{ce} = [k_e(dT_e/dy)]_{y=0}$$

the transfer by diffusion of electron thermal energy is

$$q_{de} = -F_w(Le)(1 - \alpha)(T_e)_{y=0}$$

and the energy transfer due to the gain of energy by the excessions in crossing the sheath is

$$q_{si} = -0.4F_w(Le)\alpha\gamma_1(T_e)_{y=0}$$

The dimensional heat-transfer q_w' is related to the dimensionless quantity by $q_w = q_w'L/k_\infty T_\infty$. To complete the problem formulation, the boundary conditions at the upper surface are (in dimensionless form) at $y = 1$: $T_e = T_g = 1$, $f = f_\infty$, and $dw/dy = 0$, and, in addition, at the cold surface, $w = 0$. The fluid motion (which occurs only in the presence of net current flow and a magnetic field) is assumed to be unimpeded at the upper surface.

It is rather evident from the form of the equations and the transport properties that numerical solutions are indicated. Such solutions have been obtained, although with some difficulty; space does not permit discussion of the numerical technique involved.

III. Results and Discussion‡

3.1 Discussion of Numerical Results§

3.1.1 Sheath-transition region model and energy transfer by electric fields

The effect of incorporating a transition region near the cold surface on temperature profiles and total heat transfer is shown in Fig. 3 for the case $\lambda_\infty \equiv (\lambda_{eh})_\infty = 0.140$ and $T_\infty = 50{,}000°K$. The cold ion, isothermal electron sheath model [Eqs. (36a, 36b, and 41)] has been employed. In one case (curve 2, $A_w = 1.0$) the transition region has been neglected, and the continuum equations are assumed to be valid to the sheath edge, whereas in the other case (curve 4, $A_w = 0.25$) the transition region begins when the ion drift velocity is 25% of that at the sheath edge. The differences in the two results are attributable to the fact that the transition region model employed yields less energy transfer between electrons and ions by electric fields than if the continuum equations are assumed valid to the sheath edge. It is observed that the differences are small, and it therefore may be concluded that the solution is not greatly affected by the transition region. This conclusion is further supported by the observation that the boundary condition on electron temperature diminishes in importance as thermal equilibrium is approached, and hence the effect of the transition region will diminish at higher values of λ_∞. Of course, if the value of the ion drift velocity at the transition region edge is assumed

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† It is mentioned that the results for the isothermal assumption are basically inconsistent, since collisional and transport effects were neglected in the equations of motion; hence there exists no mechanism to maintain isothermality.

‡ A more detailed presentation of the results of this investigation, including some useful analytical solutions, can be found in Ref. 12.

§ Unless otherwise specified, the following values of the parameters are employed in the numerical computations: $p = 1$ atm, $L = 10^{-3}$m, $T_w = 500°K$, $Q_0 = 2$, $\beta_{e\infty} = 0$, $\alpha = 0$. Variations in $\lambda_\infty \equiv (\lambda_{eh})_\infty$ are for parametric purposes primarily; physically, they may be construed as variations in L.

successively smaller, a point will eventually be reached where the mole fraction of ions existing at the transition region edge will be substantial compared to the mole fraction existing at the upper surface. This indicates that the spatial extent of the transition region so defined is not negligible. The resultant effect is to decrease the flux of ions toward the wall, thereby decreasing heat transfer. For example, the results of a calculation with $A_w = 0.05$, when compared with those for $A_w = 0.25$, yielded a decrease in ion flux of 14%, a decrease in heat transfer of 11%, and an increase in electron wall temperature of 13%.

The influence of the models of the sheath-transition region proposed in Sect. 2.4 have been investigated by comparing the results of the cold ion, isothermal electron model calculation at $T_\infty = 50{,}000°\mathrm{K}$, $\lambda_\infty = 0.140$, and $A_w = 0.5$ with the results of calculations at the same conditions for the completely isothermal model [Eqs. (37a, 37b, and 42)] and the adiabatic model [Eqs. (38a, 38b, 43, and 44)]. The resulting variations in heat transfer and electron wall temperature effected by these models were not great (maximum variations of 0.7 and 7.4%, respectively). It is concluded that the solution is not greatly affected by the sheath-transition region model employed. In all subsequent calculations, except for those discussed in the next paragraph, the isothermal sheath model is employed with $A_w = 0.5$.

In Fig. 3, the curves labeled 1 and 3 have been computed identically to curves 2 and 4, respectively, with the exception that energy transfer between ions and electrons by electric fields is neglected in the region external to the sheath-transition region. It is observed that this energy-transfer mechanism has an appreciable effect on the electron temperature profile and the total heat transfer to the wall. It should be noted that the major part of this electric field energy transfer occurs near the cold surface where E_y is large; this accounts for the decreased effect when a transition region is included. This observation indicates that a useful approximation in more complex problems may be to assume a value of A_w as low as is consistent with neglecting the spatial extent of the transition region and then neglect the electric field energy transfer external to the sheath. This is affirmed by the previously discussed calculation for $A_w = 0.05$ wherein the difference in total heat transfer resulting from neglecting this energy transfer was 3.6%; the fact that this value of A_w is somewhat too low to neglect the spatial extent of the transition region is of little consequence in this regard.

The preceding remarks are qualitatively applicable to different gas conditions or other noble gases. The quantitative effects are modified by the fact that, in situations where the nonequilibrium contribution of electrons to energy transfer is relatively greater, the differences resulting from the use of the various sheath-transition region models will be somewhat more pronounced.

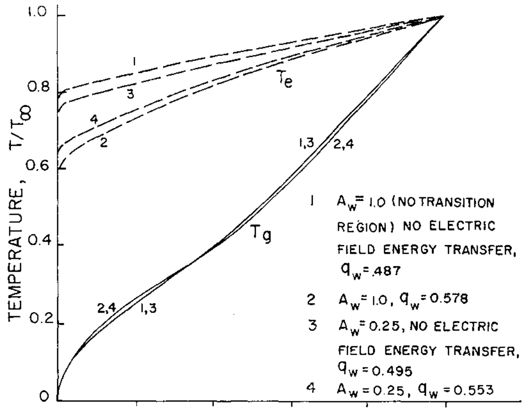

Fig. 3 Sheath-transition region and electric field energy transfer effects on temperature profiles; $T_\infty = 50{,}000°\mathrm{K}$, $\lambda_\infty = 0.140$, cold ion sheath model.

3.1.2 Thermal nonequilibrium effects on property profiles and energy transfer

The parameter that best characterizes the degree of thermal nonequilibrium is $\lambda_\infty \equiv (\lambda_{eh}')_\infty L^2/(k_e')_\infty$. It represents the ratio of the collisional energy-transfer rate between electrons and heavy particles to the energy-transfer rate by thermal conduction in electrons. If this ratio is small, the electrons are essentially thermally uncoupled, whereas if this ratio is large, thermal equilibrium prevails. The chief virtue of this parameter, as the results will show, is that the value required for virtually complete nonequilibrium is insensitive to the relative magnitudes of k_e, k_{in}, and D_A; hence, the value is not sensitive to gas conditions.

Because of the dependence of the transport properties on temperature and composition, it is necessary to appreciate the effects of thermal nonequilibrium on the temperature and composition profiles before the effects on heat transfer can be fully understood. These effects of thermal nonequilibrium, characterized by λ_∞,¶ on the temperature and composition profiles are shown in Fig. 4. The most important effect portrayed in this figure is the change in the heavy particle temperature profile as λ_∞ varies. This change in temperature profile is largely due to the change in temperature and composition dependence of the effective thermal conductivity. For example, at $\lambda_\infty = 0$, the heavy particle thermal conductivity determines T_g, and since $k_{in} \sim fT_g/[f + (1 - f)Q_0]$ exhibits a maximum, the slope of the heavy particle temperature profile exhibits a minimum. This is clearly evident in Fig. 4. On the other hand, at $\lambda_\infty = \infty$, the electron thermal conductivity is significant in determining T_g, and since $k_e \sim T_e^{5/2}$, this tends to yield a temperature profile with a continually decreasing slope; this is also evident in Fig. 4. The net result of this effect is to decrease the

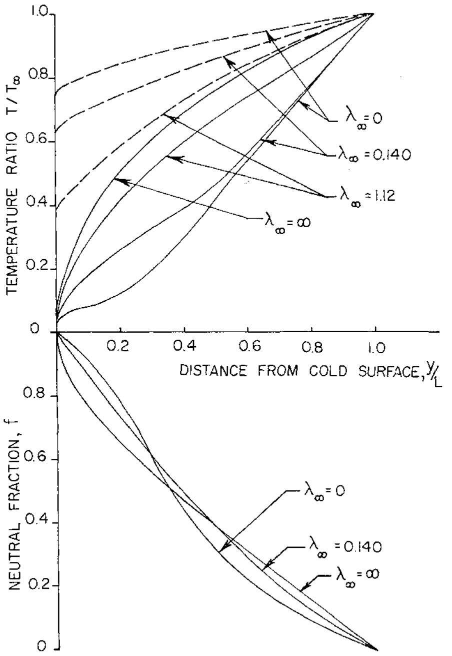

Fig. 4 Thermal nonequilibrium effects on temperature and composition profiles; $T_\infty = 50{,}000°\mathrm{K}$.

¶ The parameter λ_∞ employed in the figures is actually $(\lambda_{eh}')_\infty L^2/(k_e' + k_{in}')_\infty$. This differs from the parameter λ_∞ discussed in the text by the factor $(k_e' + k_{in}')_\infty/(k_e')_\infty$; however, for all cases considered here, this factor is 1.04, and hence the difference in the two parameters is not significant.

heavy particle temperature everywhere as thermal nonequilibrium becomes more pronounced. As will be seen, since both k_{in} and D_A are functions of T_g, this change plays an important role in determining the magnitude of thermal nonequilibrium effects on total heat transfer.

Two additional features evident in Fig. 4 are worthy of note. First, the electron temperature is not constant between the plates when $\lambda_\infty = 0$. This is, of course, due to the energy transfer by electric fields between electrons and ions. As mentioned in Sec. 3.1.1, a minor portion of this energy transfer occurs external to the sheath-transition region, whereas the major portion occurs in this region (evidenced by the boundary condition on electron temperature). Second, the region of thermal nonequilibrium is not well confined near the cold surface at higher values of λ_∞. This is due to the strong temperature dependences of the collisional energy coupling and the thermal conductivities. It is remarked in passing that calculations performed for $Q_0 = 1$, resulting in a factor of 2 increase in D_A, indicate that the influence of thermal nonequilibrium on the temperature profiles in Fig. 4 is not qualitatively altered. The major effect of the increase in D_A is a decrease in electron wall temperature in the nonequilibrium situation due to increased electric field energy transfer between electrons and ions. Finally, it is noted that the effects exhibited in Fig. 4 are not expected to be altered in the noble gases because of the previously mentioned similarity in temperature-composition dependence of the transport properties.

The most important effects of thermal nonequilibrium on diffusional processes, as indicated in Fig. 5, are twofold. First, the heat transfer due to recombination at the cold surface (q_{di}) is essentially a constant fraction of the total heat transfer, regardless of the degree of nonequilibrium. This reflects the fact that $q_{di} \sim D_A \sim T_g$. As observed in the preceding discussion, T_g decreases as λ_∞ decreases; hence q_{di} does likewise. Second, the heat transfer due to the diffusion of electron thermal energy (q_{de}), as well as the related heat conduction, becomes an appreciable part of the total heat transfer as complete nonequilibrium is approached. It should be noted however, that the total heat transfer decreases as $\lambda_\infty \rightarrow 0$, as shown in Fig. 6. This indicates that the tendency for diffusion to increase the electronic contribution to heat transfer in the nonequilibrium condition is not sufficient to overcome the factors tending to decrease this heat transfer. These factors are of course the loss of the electronic contribution to conduction in the equilibrium condition, the decrease in recombinational heat transfer, and finally, the decrease in heavy particle conduction as complete nonequilibrium is approached. This last effect is again due to the decrease in T_g as λ_∞ decreases, since $q_{ci} \sim k_{in} \sim T_g$.

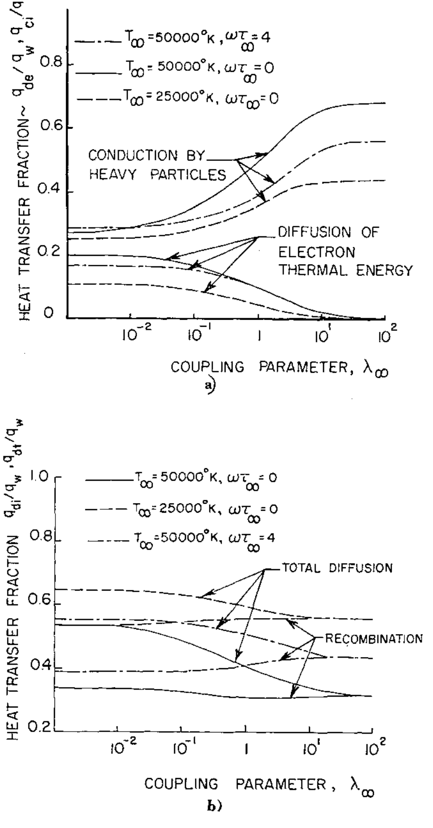

Fig. 5 Thermal nonequilibrium effects on heat transfer due to various sources. (The heat transfer due to electronic conduction is indistinguishable from that due to diffusion of electron thermal energy.)

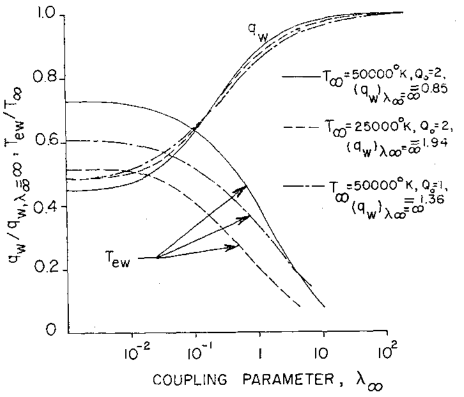

Fig. 6 Thermal nonequilibrium effects on total heat transfer and electron temperature.

The relative roles of the conduction and diffusion processes in both electrons and heavy particles in the presence of thermal nonequilibrium effects can be more fully appreciated by examining the results for two different upper surface temperatures. These results are also shown in Figs. 5 and 6. Because of the temperature dependences of the transport properties ($k_e \sim T_e^{5/2}$, $k_{in} \sim T_g$, $\rho D_A \sim T_g$), it is to be expected that the electronic contribution to heat transfer will be relatively less at a lower temperature level, while the contribution of recombination will be relatively greater. The lesser contribution of the electrons at lower-temperatures is evidenced in Fig. 5 by both the decreased fraction of the total heat transfer, which is directly attributable to the electronic component, and the decreased effect of nonequilibrium on the heat transfer due to heavy particle conduction. The greater contribution of recombination to the total heat transfer at the lower temperature is also evident in Fig. 5. Superficially, it might be expected that thermal nonequilibrium effects on total heat transfer would be considerably less at lower temperatures because of the decreased contribution of electrons. Figure 6 indicates, however, that nonequilibrium effects are only slightly less at $T_\infty = 25{,}000°K$ than at $T_\infty = 50{,}000°K$. This is of course due to the increased importance of recombinational heat transfer at the lower temperature and the fact that the appreciable effect of nonequilibrium on this quantity is not very sensitive to these temperature levels. It is also worthy of note that the fractional change in total heat transfer from equilibrium to complete nonequilibrium is only slightly less when the diffusion coefficient is doubled (Fig. 6): again, this is primarily because of the insensitivity to nonequilibrium effects of the fraction of the total heat transfer due to recombination.

However, at temperature levels where the gas at the hot surface is no longer fully ionized, the recombinational energy transfer becomes much less sensitive to nonequilibrium effects. This is due to the fact that the variation of $k_{in}(T_g, f)$ is such that the nonequilibrium heavy particle temperature differs much less from the equilibrium profile than at higher temperatures. Hence, it is expected that, at these conditions, the total heat transfer is much less sensitive to nonequilibrium effects. This has been verified by a computation at $T_\infty = 15{,}000°K$ where $f_\infty = 0.45$, which yielded a decrease in total heat transfer from equilibrium to complete nonequilibrium of only 7.7%. This result is in qualitative agreement with the result obtained by Camac and Kemp[1] in argon.

Finally, it should be noted in Fig. 6 that the values of λ_∞, which are indicative of transition from thermal nonequilibrium to equilibrium, are not sensitive to temperature level or the magnitude of the ion-neutral diffusion coefficient. Specifically, the value of λ_∞ required for a change in total heat transfer (from nonequilibrium toward equilibrium) of 90% of the total change is in the range 2.8–4.4, whereas the value for a similar change of 10% is in the range 0.02–0.03, with the higher value occurring at the lower temperature. It is also remarked that calculations indicate that the presence of a magnetic field, which significantly alters the relative importance of the electronic thermal conductivity, does not have a significant effect on the transitional values of λ_∞.

The preceding discussion allows two general conclusions with regard to thermal nonequilibrium effect on total heat transfer in gases other than hydrogen. First, for gases in which the electronic contribution to the thermal conductivity is greater relative to the heavy particle contribution and the contribution from diffusional processes, thermal nonequilibrium will have a greater effect in reducing heat transfer. Generally, in any heavier gas at the same temperature and ionization level as hydrogen, the relative electronic contribution will be greater. This is confirmed in the specific case of argon by the brief discussion of transport properties given in Sec. 2.3. The second general conclusion is that the values of λ_∞ indicative of transition from thermal equilibrium to nonequilibrium are not appreciably different for other gases. Both of these conclusions of course presuppose that the temperature-composition dependences of k_{in} and D_A in other gases are similar to those in hydrogen, and, hence, these conclusions are limited to the noble gases.

Before terminating this discussion of thermal nonequilibrium effects on energy transfer, it should be pointed out that another possible approximation to the chemical state of the gas, not treated here, is that where the ion and electron densities are in chemical equilibrium with the neutrals at the electron temperature. This approximation may be valid at sufficiently high temperatures, and, for the present cases, would yield a fully ionized gas everywhere, except for a thin layer near the cold wall, for electron temperatures above 25,000°K. In this case, diffusional processes and thermal conduction by neutrals would play a very minor role in determining the total energy transfer, and thermal nonequilibrium would have a considerably more drastic effect in reducing heat transfer.

3.1.3 Magnetic field effects on heat transfer

In the absence of a net current passing between the plates, it is evident from the governing equations and transport properties that the sole effect of a magnetic field is to decrease the electronic thermal conductivity. It is to be expected then that the magnetic field will decrease the total heat transfer in both equilibrium and nonequilibrium situations; this is clearly evident in the numerical results shown in Fig. 7. These results also indicate that nonequilibrium effects on total heat transfer are less in the presence of a magnetic field; this merely reflects the fact that the effects of thermal equilibrium on heat transfer decrease when the electronic contributions decrease. Further, the previous discussion of thermal nonequilibrium effects on heat transfer has indicated that the electronic contributions are greater at equilibrium than at nonequilibrium. In the presence of a magnetic field, this fact is evidenced in Fig. 5 by the increased fraction of the total heat transfer due to recombination at equilibrium relative to that at nonequilibrium. Hence, it is expected that the effect of the magnetic field in decreasing heat transfer would be less as the amount of thermal nonequilibrium increases; this feature is also evident in Fig. 7.

It is of interest to compare the numerical results of Fig. 7 with the effect of the magnetic field on heat transfer if electronic and ionic heat conduction were the only processes of importance. This has been accomplished by means of analytical solutions for the case of a fully ionized gas in thermal equilibrium, in which no neutrals are present and diffusion vanishes. Such a situation could be envisioned if the cold surface were noncatalytic to recombination. Assuming that $k_e \sim T_g^{5/2}$, $k_i \sim T_g^{5/2}$, and $(\omega\tau)_e \sim T_g^n$, the governing equations can readily be solved for $n = 0, 1.75$, and 2.33. These results are shown in Fig. 7 for $k_e/k_i = 25$, which is the value appropriate for fully ionized hydrogen. Comparison of the results for $n = 0$ and $n = 2.33$ shows the considerable effect of the temperature dependence of $(\omega\tau)_e$ in reducing the effects of the magnetic field on heat transfer. Since the actual dependence of $(\omega\tau)_e$ is essentially $T_g^{5/2}$, this effect should be slightly more pronounced in the actual case than the two curves indicate. The differences between the analytical result for $n = 2.33$ and the actual numerical results indicate the importance of the other energy-transfer mechanisms, notably diffusion and heavy particle conduction, in determining the total heat transfer.

The point deserving emphasis in this discussion is that the large effect of the magnetic field in reducing the electronic thermal conductivity at the hot surface does not have a correspondingly large influence on total heat transfer; this is, of course, partly due to sources of heat transfer other than electronic conduction and partly due to the strong temperature dependence of $(\omega\tau)_e$. This observation must be tempered by the fact that, for other gases where the contribution of the electronic thermal conductivity to the total heat transfer is a greater fraction than in hydrogen, the effect of the magnetic field will be correspondingly greater.

3.1.4 Effect of net current on electrical characteristics

The effect of passing a net current between the plates on various potential differences and on the electron wall temperature is shown in Fig. 8 for the complete nonequilibrium conditions ($\lambda_\infty = 0$). It should be noted that the current consists of an excess ion flux to the cold surface, which tends to repel more electrons as it is made more negative with

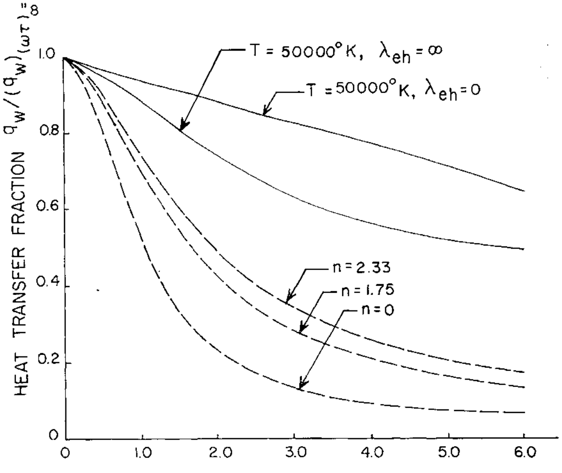

Fig. 7 Magnetic field effects on heat transfer. Broken lines are analytical results for a fully ionized gas (noncatalytic wall) assuming $(\omega\tau)_e \sim T_e^n$ and $r = k_e/k_{in} = 25$.

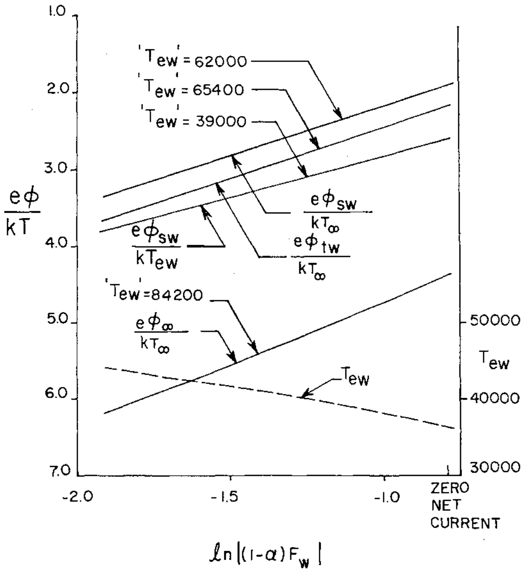

Fig. 8 Effect of net current on various potential differences; $T\infty = 50{,}000°\text{K}$.

respect to the hot wall, thus accounting for the increase in T_{ew} as the current increases. By conventional Langmuir probe theory, in which both the electron temperature at the sheath edge and the total ion flux to the cold surface are assumed to be unaffected by the net current, the slopes of the curves labeled $e\phi_{sw}/kT_\infty$ and $e\phi_{sw}/kT_{ew}$ in Fig. 8 should be equal to T_{ew}/T_∞ and unity, respectively. This allows the values for T_{ew}, indicated on each curve, to be deduced from these slopes (evaluated at the point of zero net current). The fact that the temperature deduced from $e\phi_{sw}/kT_{ew}$ is not equal to that of the electrons at the sheath edge indicates that the ion flux is not independent of net current (the ion flux increases by 6.7% as the current increases over the range shown in Fig. 8). The temperature deduced from $e\phi_{sw}/kT_\infty$ (62,000°K) differs drastically from the actual value at the sheath edge (36,000°K) which is somewhat surprising in view of the rather modest change in T_{ew} with j. The rather obvious significance of this discussion is that interpretation of a conventional Langmuir probe characteristic in terms of the electron temperature at these conditions is highly unreliable, unless the voltage is varied at a rate that allows insufficient time for the electron temperature at the sheath edge to change.

3.2 Conclutions

The major conclusions that can be drawn from the present investigation are:

1) The gross features of the continuum solution are relatively insensitive to the sheath-transition region model employed. The energy transfer between electrons and ions by electric fields external to the sheath-transition region is sensitive to the ion diffusion velocity at the edge of the transition region and, in general, may not be neglected.

2) The ratio $(\lambda_E')_\infty = \lambda_\infty' L^2/(k_e')_\infty$ is a valid indicator of thermal nonequilibrium effects. In the cases studied, $(\lambda_E)_\infty \lesssim 0.02$ resulted in essentially no collisional energy transfer between electrons and heavy particles, whereas $(\lambda_E)_\infty \gtrsim 4.0$ resulted in virtual thermal equilibrium.

3) Thermal nonequilibrium effects on total heat transfer are strongly influenced by the relative importance of electronic heat conduction to the other heat-transfer mechanisms and, in addition, by the temperature and composition dependences of the heavy particle thermal conductivity and the ion-neutral diffusion coefficient.

4) Magnetic field effects on total heat transfer are much less than the effect on the electron thermal conductivity; departures from thermal equilibrium result in further decreased magnetic field effects.

5) A simple interpretation of the conventional Langmuir probe characteristic in terms of electron temperature at the sheath edge is not possible.

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